Reg. No. | | | | | | | | |

IV Semester B.Sc. Degree Examination, August/September - 2023

MATHEMATICS

(CBCS Scheme Repeaters 2020 Onwards)

Paper : IV

Time : 3 Hours

Maximum Marks : 70

Instructions to Candidates :

Answer ALL questions

I. Answer any **FIVE** questions. **(5×2=10)**

a. Define a normal Subgroup.

b. Let $f:G\to G'$ be an isomorphism, then show that $f(a^{-1})=[f(a)]^{-1},\ \forall\ a\in G$

c. Write the Fourier coefficients of the Fourier Series of *f(x)* in the interval $(-\pi,\pi)$

d. State Rolle's theorem.

e. Evaluate $\lim\limits_{x\to 0}\left(\dfrac{x-\sin x}{x^2}\right)$

f. State Taylor's theorem for a function of two variables.

g. Find the Particular integral of the equation $(D^3+D^2-D-1)y=e^x$

h. Show that the equation $x^2y''+3xy'+y=0$ is exact

II. Answer any **TWO** questions **(2×5=10)**

a) Prove that a Subgroup H of a group G is normal if and only if $gHg^{-1}\in H\,\forall g\in G$ and $\forall\ h\in H$.

b) $f:(Z_8,+_8)\to(Z_2,+_2)$ is given by *f(x)* = *r where r is the remainder when x is divided by 2. Show that f is a homomorphism. Find Kerf.*

c) Let $S=\{1,2,3,4,5\}$ and $f=\begin{pmatrix}1&2&3&4&5\\1&5&3&2&4\end{pmatrix}$, $g=\begin{pmatrix}1&2&3&4&5\\2&3&1&5&4\end{pmatrix}$

Show that $fog\neq gof$

[P.T.O.

III. Answer any **TWO** questions. **(2×5=10)**

a) Obtain the Fourier series for the function $f(x)=x^2$ over the interval $(-\pi,\pi)$ and hence deduce that

$$\frac{\pi^2}{12}=\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}----$$

b) Obtain the Fourier series for the function $f(x)=\begin{cases}1+\frac{4x}{3}\ in\ \frac{-3}{2}<x\le 0\\ 1-\frac{4x}{3}\ in\ 0\le x<\frac{3}{2}\end{cases}$ Hence

deduce that $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+----$.

c) Find the Fourier half range Sine Series for the function f defined by $f(x)=1$ in $(0,\pi)$

IV. Answer any **THREE** questions : **(3×5=15)**

a) Discuss the continuity of the function $f(x)=\begin{cases}1+x & for\ \ x\le 2\\ 5-x & for\ \ x\ge 2\end{cases}$ at $x=2$

b) State and prove Lagrange's Mean value theorem.

c) Verify the Cauchy's Mean Value theorem of the function

$$f(x)=\sin x\ and\ g(x)=\cos x\ in\left[\frac{-\pi}{2},0\right].$$

d) Expand $e^{ax}\cos(by)$ in Taylor's series upto second degree terms about the origin.

e) Find the extreme value of the function $f(x,y)=x^3+y^3-3x-12y+20$.

V. Answer any **THREE** questions : **(3×5=15)**

a) Solve : $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=\cos 3x$

b) Solve : $(x+2)^2y''-(x+2)y'+y=3x+4$

c) Solve : $\frac{d^2y}{dx^2}-(\cot x)\frac{dy}{dx}-(1-\cot x)y=e^x\sin x$

d) Solve the system of equations: $\frac{dx}{dt}=3x-4y$; $\frac{dy}{dt}=x-y$

e) Solve by the method of variation of parameters : $\frac{d^2y}{dx^2}-y=\frac{2}{1+e^x}$.

VI. Answer any **TWO** questions : **(2×5=10)**

a) When a Sinusoidal Voltage E Sin(wt) is passed through a half-wave rectifier which clips the negative portion of the Wave, the resulting periodic function is given by

$$u(t)=\begin{cases}0 & for\ \frac{-\pi}{\omega}<t<0\\ E\,\text{Sin}\,(\omega t)\ for\ 0<t<\frac{\pi}{\omega}\end{cases}$$

Develop this function in a Fourier Series.

b) An alternating current after passing through a rectifier has the form $I=\begin{cases}I_0 \sin\theta & for\ 0<\theta\leq\pi.\\ 0 & for\ \pi<\theta\leq 2\pi.\end{cases}$ where I_o is the maximum current Express I as Fourier Series in $(0,2\pi)$

c) If F(t) is the Periodic function and is defined in One period as $F(t)=|t|$ for $-\pi\leq t\leq\pi$, find the solution of the differential equation $y''-y = F(t)$.
