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Reg. No. [ ]

VI Semester B.Sc. Degree Examination, August/September - 2023
MATHEMATICS
(CBCS Scheme)
Paper : VII

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates :

Answer **ALL** questions.

I. Answer any **FIVE** questions. **(5×2=10)**

1. Prove that the set $S=\{(3,2,-1),(0,4,5),(6,4,-2)\}$ is linearly dependent in $V_3(R)$.

2. Find the matrix of linear transformation $T:V_2(R)\to V_2(R)$ defined by $T(x,y)=(x,-y)$ with respect to the standard bases.

3. Write the relation between the cartesian Co-ordinates and cylindrical Co-ordinates of a point.

4. Prove that in spherical Co-ordinate system $\hat{e}_r\times\hat{e}_\theta=\hat{e}_\phi$.

5. Verify the integrability condition for $(y+z)dx+(x+z)dy+(x+y)dz=0$.

6. Solve $\frac{xdx}{y^2z}=\frac{dy}{zx}=\frac{dz}{y^2}$.

7. Form the partial differential equation by eliminating the arbitrary function 'f' from $z=f\left(x^2-y^2\right)$.

8. Solve $\left(D^2-4DD'+4D'^2\right)z=0$.

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II. Answer any **Three** questions. **(3×5=15)**

9. Prove that "The intersection of any two subspaces of a vector space V(F) is also a sub-space of V(F).

10. Find the dimension and basis of the subspace spanned by the vectors $\{(2, 4, 2), (1, -1, 0), (1, 2, 1), (0, 3, 1)\}$ of $V_3(R)$.

11. Find the matrix of linear transformation $T : R^2 \to R^2$ defined by $T(x, y) = (x + 4y, 2x - 3y)$ relative to the bases $B_1 = \{(1, 0), (0, 1)\}, B_2 = \{(1, 3), (2, 5)\}$.

12. State and prove rank-nullity theorem.

13. Show that the set of all eigen vectors associated with the eigen value λ of a linear transformation T together with zero vectors is a subspace of the vectorspace.

III. Answer any **Three** questions. **(3×5=15)**

14. Show that the spherical Co-ordinate system is orthogonal curvilinear co-ordinate system.

15. Express the vector $\vec{f} = 2x\hat{i} - 2y^2\hat{j} + xz\hat{k}$ in cylindrical Co-ordinates and find f_ρ, f_ϕ, f_z

16. Express $\vec{f} = z\hat{i} - 2x\hat{j} + y\hat{k}$ in spherical polar Co-ordinates and hence find f_r, f_θ, f_ϕ.

17. Solve $\frac{\partial u}{\partial t} = 16\frac{\partial^2 u}{\partial x^2}$ given

$$u(0, t) = 0,\ u(1, t) = 0,\ \forall t$$

$$u(x, 0) = x^2 - x,\ 0 \le x \le 1$$

18. Solve

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2} \text{ given } u(0, t) = 0,\ u(l, t) = 0,\ u(x, 0) = a\,\mathrm{Sin}\left(\frac{\pi x}{l}\right) \text{ and } \left(\frac{\partial u}{\partial t}\right)_{t=0} = 0$$

IV. Answer any **Three** questions. **(3×5=15)**

19. Verify the condition of integrability and solve $3x^2dx + 3y^2dy - \left(x^3 + y^3 + e^{2z}\right)dz = 0$.

20. Solve $\dfrac{dx}{x(y-z)} = \dfrac{dy}{y(z-x)} = \dfrac{dz}{z(x-y)}$.

21. Form the partial differential equation by eliminating arbitrary function ϕ from $lx + my + nz = \phi\left(x^2 + y^2 + z^2\right)$.

22. Solve $p^3 + q^3 = 27z$.

23. Find the complete integral of $z^2\left(p^2 + q^2 + 1\right) = 1$ by using charpit's method.

V. Answer any **Three** questions. **(3×5=15)**

24. Find a linear transformation $T : R^2 \rightarrow R^2$ such that $T(1, 0) = (1, 1)$ and $T(0, 1) = (-1, 2)$ prove that T maps the square with vertices $(0, 0)$, $(1, 0)$, $(1, 1)$ and $(0, 1)$ into parallelogram.

25. The vibration of an elastic string is governed by $pde \dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2}$. The length of the string is π and ends are fixed. The initial velocity is zero and the initial deflection is $u(x, 0) = 2(\text{Sin } x + \text{Sin } 3x)$. Find the deflection $u(x, t)$ of the vibrating string for $t > 0$.

26. Evaluate $\iiint_v \left(x^2 + y^2 + z^2\right) dxdydz$ where v is the sphere having centre at the region and radius equal to 'a' by changing the variable to spherical polar Co-ordinates.

27. Find the curves which satisfy the differential equation $ydx + zdy - ydy + xdz = 0$ and which lie on the plane $2x - y - z = 1$.

28. Reduce the equation $r+2s+t=0$ to canonical form.