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Reg.No.

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VI Semester B.Sc. Degree Examination, August/September - 2023

MATHEMATICS

(CBCS Scheme)

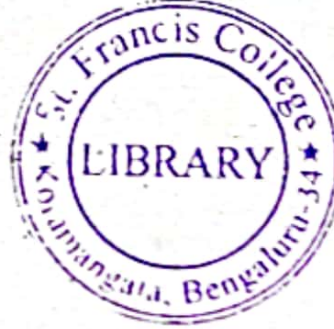
Paper : VII

Time : 3 Hours

Maximum Marks : 70

Instructions to Candidates :

Answer ALL questions.



I. Answer any FIVE questions.

(5×2=10)

1. Prove that the set $S = \{(3, 2, -1), (0, 4, 5), (6, 4, -2)\}$ is linearly dependent in $V_3(R)$.
2. Find the matrix of linear transformation $T: V_2(R) \rightarrow V_2(R)$ defined by $T(x, y) = (x, -y)$ with respect to the standard bases.
3. Write the relation between the cartesian Co-ordinates and cylindrical Co-ordinates of a point.
4. Prove that in spherical Co-ordinate system $\hat{e}_r \times \hat{e}_\theta = \hat{e}_\phi$.
5. Verify the integrability condition for $(y+z)dx + (x+z)dy + (x+y)dz = 0$.
6. Solve $\frac{xdx}{y^2z} = \frac{dy}{zx} = \frac{dz}{y^2}$.
7. Form the partial differential equation by eliminating the arbitrary function 'f' from $z = f(x^2 - y^2)$.
8. Solve $(D^2 - 4DD' + 4D'^2)z = 0$.

[P.T.O.]



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(3×5=15)

II. Answer any Three questions.

9. Prove that "The intersection of any two subspaces of a vector space $V(F)$ is also a sub-space of $V(F)$."
10. Find the dimension and basis of the subspace spanned by the vectors $\{(2, 4, 2), (1, -1, 0), (1, 2, 1), (0, 3, 1)\}$ of $V_3(R)$.
11. Find the matrix of linear transformation $T: R^2 \rightarrow R^2$ defined by $T(x, y) = (x + 4y, 2x - 3y)$ relative to the bases $B_1 = \{(1, 0), (0, 1)\}$, $B_2 = \{(1, 3), (2, 5)\}$.
12. State and prove rank-nullity theorem.
13. Show that the set of all eigen vectors associated with the eigen value λ of a linear transformation T together with zero vectors is a subspace of the vectorspace.

III. Answer any Three questions.

(3×5=15)

14. Show that the spherical Co-ordinate system is orthogonal curvilinear co-ordinate system.
15. Express the vector $\vec{f} = 2x\hat{i} - 2y^2\hat{j} + xz\hat{k}$ in cylindrical Co-ordinates and find f_ρ, f_ϕ, f_z .
16. Express $\vec{f} = z\hat{i} - 2x\hat{j} + y\hat{k}$ in spherical polar Co-ordinates and hence find f_r, f_θ, f_ϕ .
17. Solve $\frac{\partial u}{\partial t} = 16 \frac{\partial^2 u}{\partial x^2}$ given
 $u(0, t) = 0, u(1, t) = 0, \forall t$
 $u(x, 0) = x^2 - x, 0 \leq x \leq 1$

18. Solve

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \text{ given } u(0, t) = 0, u(l, t) = 0, u(x, 0) = a \sin\left(\frac{\pi x}{l}\right) \text{ and } \left(\frac{\partial u}{\partial t}\right)_{t=0} = 0$$



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(3×5=15)

IV. Answer any Three questions.

19. Verify the condition of integrability and solve $3x^2 dx + 3y^2 dy - (x^3 + y^3 + e^{2z}) dz = 0$.

20. Solve $\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$.

21. Form the partial differential equation by eliminating arbitrary function ϕ from $lx + my + nz = \phi(x^2 + y^2 + z^2)$.

22. Solve $p^3 + q^3 = 27z$.

23. Find the complete integral of $z^2(p^2 + q^2 + 1) = 1$ by using charpit's method.

V. Answer any Three questions.

(3×5=15)

24. Find a linear transformation $T: R^2 \rightarrow R^2$ such that $T(1, 0) = (1, 1)$ and $T(0, 1) = (-1, 2)$ prove that T maps the square with vertices $(0, 0)$, $(1, 0)$, $(1, 1)$ and $(0, 1)$ into parallelogram.

25. The vibration of an elastic string is governed by $pde \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$. The length of the string is π and ends are fixed. The initial velocity is zero and the initial deflection is $u(x, 0) = 2(\sin x + \sin 3x)$. Find the deflection $u(x, t)$ of the vibrating string for $t > 0$.

26. Evaluate $\iiint_v (x^2 + y^2 + z^2) dx dy dz$ where v is the sphere having centre at the origin and radius equal to 'a' by changing the variable to spherical polar Co-ordinates.

27. Find the curves which satisfy the differential equation $ydx + zdy - ydy + xdz = 0$ and which lie on the plane $2x - y - z = 1$.

28. Reduce the equation $r+2s+t=0$ to canonical form.