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I Semester B.Sc. Degree Examination, August - 2021

MATHEMATICS

Mathematics - I

(CBCS Semester Scheme)

Time : 3 Hours

Maximum Marks : 70

Instructions to Candidates : Answer all questions.

I. Answer any **Five** questions.

(5×2=10)

- 1) Define rank of a matrix A of order $m \times n$.
- 2) Show that the system of equations $x + 2y + z = 0$, $x - 2z = 0$, $2x + y - 3z = 0$ has only trivial solution.
- 3) Find the n^{th} derivative of $\sin^2 x$.
- 4) If $u = x^2 y$, prove that $\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial^2 u}{\partial x \partial y}$.
- 5) Evaluate : $\int_0^{\frac{\pi}{2}} \sin^8 x \, dx$.
- 6) Evaluate : $\int_0^{\frac{\pi}{2}} \sin^6 x \cos^5 x \, dx$.
- 7) Find the value of k such that the spheres $x^2 + y^2 + z^2 + 4x + ky + 2z + 6 = 0$ and $x^2 + y^2 + z^2 + 2x - 4y - 2z + 6 = 0$ are orthogonal.
- 8) Find the equation of the right circular cone whose vertex is (1,2,3), axis is the y - axis and semi vertical angle is 30° .



[P.T.O.]



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II. Answer any **Three** questions.

(3×5=15)

- 9) Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \end{bmatrix}$ by reducing to normal form.
- 10) Test the system of equations $x+2y-z=3$, $3x-y+2z=1$, $2x-2y+3z=2$ for consistency and solve if possible.
- 11) Find the eigen values and corresponding eigen vectors of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$.
- 12) State and prove Cayley - Hamilton Theorem.
- 13) By using Cayley - Hamilton Theorem, find the inverse of the matrix $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$.

III. Answer any **Three** questions.

(3×5=15)

- 14) Find the n^{th} derivative of $\frac{4x}{(x+1)^2(x-1)}$.
- 15) If $y = e^{m \sin^{-1} x}$ prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$.
- 16) If $Z = \sin(ax+y) + \cos(ax-y)$, prove that $\frac{\partial^2 Z}{\partial x^2} = a^2 \frac{\partial^2 Z}{\partial y^2}$.
- 17) State and prove Euler's Theorem for a homogeneous function of two variables.
- 18) If $x = r \cos \theta$, $y = r \sin \theta$, find $J = \frac{\partial(x,y)}{\partial(r,\theta)}$ and $J' = \frac{\partial(r,\theta)}{\partial(x,y)}$. Also verify $JJ' = 1$.

IV. Answer any **Two** questions.

(2×5=10)

- 19) Obtain the reduction formula for $\int \cos^n x \, dx$.

20) Evaluate : $\int_0^{\infty} \frac{x^2}{(1+x^2)^3} dx$.

21) Using Leibnitz's rule, evaluate $\int_0^{\infty} e^{-x} \frac{\sin \alpha x}{x} dx$, where α is a parameter.





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V. Answer any **Two** questions.

(2×5=10)

- 22) Obtain the equation of the sphere which passes through the points $(1,0,0)$, $(0,1,0)$, $(0,0,1)$ and having its centre on the plane $3x - y + z = 2$.
- 23) Find the equation of the right circular cone which passes through the point $(1,1,2)$, has its vertex at the origin and its axis is $\frac{x}{2} = \frac{-y}{4} = \frac{z}{3}$.
- 24) Find the equation of the right circular cylinder of radius 2 units and whose axis is $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z-2}{5}$.

VI. Answer any **Two** questions.

(2×5=10)

- 25) Suppose in a stationery shop books, pens and pencils are sold with 3 books, 2 pens and 1 pencil cost Rs. 140, 2 books, 2 pens and 2 pencils cost Rs. 170, 1 book, 3 pens and 2 pencils cost Rs. 180. Find whether each item has a fixed price or each item has different prices or it is not possible to find the price of the items.
- 26) The population grows at the rate of 5% per year. How long does it take for the population to double?
- 27) A particle of mass 3 units is moving along the space curve defined by $\vec{r} = (4t^2 + t^3)\hat{i} + 5t\hat{j} + (t^3 + 2)\hat{k}$. Find
- The momentum and
 - Force acting on it at $t = 2$.

